Does it Pay to Get a Reverse Mortgage?

Valentina Michelangeli

Boston University

Institute on Computational Economics
University of Chicago
Wednesday, August 6, 2008
Does it Pay to Get a Reverse Mortgage?
Structural Estimation of Life-Cycle Dynamic Problem with Continuous State Variable

- Relevant topic in finance, public economics, IO, marketing
- Computationally challenging

I solved this model with AMPL:

- Easy: reduction in coding errors
- Fast: between 10 and 60 minutes to estimate a dynamic structural model with more than 70,000 variables
- Precise: based on mathematics
Simple Life-Cycle Model: One continuous state variable

- Backward Solution for the True Value Function

The last period value function is known and equal to $V_T(W)$

In periods $t = 1... (T - 1)$ the Bellman equation is:

$$V_t(W) = \max_c (u(c)) + \beta EV_{t+1}(W - c)$$

Given $V_{t+1}$, the Bellman equation implies, for each wealth level $W$, three equations that determine the optimal consumption, $c^*$, $V_t(W)$, and $V'_t(W)$:

- Bellman Equation: $V_t(W) = u(c^*) + \beta V_{t+1}(RW - c^*)$
- Euler Equation: $u'(c^*) - \beta RV'_{t+1}(RW - c^*) = 0$
- Envelope Condition: $V'(W) = \beta RV'(RW - c^*)$
Backward Solution for the Approximate Value Function

- Choose a functional form and a finite grid of wealth levels

Time $t$ value function is approximated by

$$V_t(W) = \Phi(W; a_t, \overline{W}_t) = \sum_{k=0}^{7} a_{k+1,t}(W - \overline{W}_t)^k$$

- We would like to find coefficients $a_t$ such that each time $t$

Bellman equation, along with the Euler and Envelope

conditions, holds with the $\Phi$ approximation

$$\Phi(W; a_t) = \max_c (u(c)) + \beta \Phi_{t+1}(W - c; a_{t+1})$$
Define three set of errors, $\lambda^b_t \geq 0, \lambda^e_{i,t} \geq 0, \lambda^\text{env}_t \geq 0$, that satisfy the following inequalities

- **Bellman Error**

\[-\lambda^b_t \leq \Phi(W_{i,t}; a_t) - [u(c^*_{i,t}) + \beta \Phi_{t+1}(RW_{i,t} - c^*_{i,t}; a_{t+1})] \leq \lambda^b_t\]

- **Euler Error**

\[-\lambda^e_{i,t} \leq u'(c^*_{i,t}) - \beta R \Phi'(RW_{i,t} - c^*_{i,t}; a_{t+1}) \leq \lambda^e_{i,t}\]

- **Envelope Error**

\[-\lambda^\text{env}_t \leq \Phi'(W_{i,t}; a_t) - \beta \Phi'_{t+1}(RW_{i,t} - c^*_{i,t}; a_{t+1}) \leq \lambda^\text{env}_t\]
Dynamic Programming
with Approximation of the Value Function

- Minimize the sum of the errors:

\[
\text{Minimize } \sum_t \sum_i \lambda_{i,t}^e + \sum_t \lambda_t^b + \sum_t \lambda_t^{env}
\]

subject to:
- Bellman error
- Euler error
- Envelope error
Dynamic Programming with Approximation of the Value Function in AMPL

- Parameters: $W_{i,t}, \beta, R$
- Variables of Interest: $c_{i,t}, a_{k,t}, \lambda^b_t, \lambda^e_t, \lambda^{env}_t$
- Other Variables: $u_{i,t}, u_{c;i,t}, W_{i,t}^+, \Phi_{i,t}, \Phi_W; i, t, \Phi^+_i, \Phi^+_W$

The DP problem can be written as:

$$\text{Minimize} \quad \sum_t \sum_i \lambda^e_{i,t} + \sum_t \lambda^b_t + \sum_t \lambda^{env}_t$$

subject to:
- Bellman Error
  $$-\lambda^b_t \leq \Phi_{i,t} - [u_{i,t} + \beta \Phi^+_i, t] \leq \lambda^b_t$$
- Euler Error
  $$-\lambda^e_{i,t} \leq u_{c;i,t} - \beta R \Phi^+_W; i, t \leq \lambda^e_{i,t}$$
- Envelope Error
  $$-\lambda^{env}_t \leq \Phi^+_W; i, t - \beta \Phi^+_W; i, t \leq \lambda^{env}_t$$
param nT:=3; #number of time periods
param r:=1.04; #interest rate
param β:=0.96; #discount factor

# Chebychev nodes
param xmin {1..nT};
param xmax {it in 1..nT};
param pi:=3.14159265;
param nx:=28; # number of nodes
param rcheb {i in 1..nx}:=(-cos((2*i-1)*pi/(2*nx)))/cos(((2*nx-1)/2*nx)*pi); #roots
param x {i in 1..nx, it in 1..nT }:=0.5*(rcheb[i]+1)*(xmax[it]-xmin[it])+xmin[it]; #nodes
param xbar {it in 1..nT}:=(xmin[it]+xmax[it])/2;

param nK:=8; #number of points in the polynomial
var ax {ik in 1..nK, it in 1..nT-1}; #polynomial coefficients
var cons {ix in 1..nx, it in 1..nT-1}>=0.001; #consumption
var VV1 {ix in 1..nx, it in 2..nT}; #next-period value function
var VVprime1 {ix in 1..nx, it in 1..nT}; #derivative of next-period value function
var u {ix in 1..nx, it in 1..nT-1}:= log(1+cons[ix, it]); #utility function
var uprime {ix in 1..nx, it in 1..nT-1}:=1/(1+cons[ix, it]); #derivative of utility function
# polynomials
# time t
param phi {ik in 1..nK, ix in 1..nx, it in 1..nT-1} = (x[ix,it]-xbar[it])^(ik-1); # basis
var poly {ix in 1..nx, it in 1..nT-1} = sum {ik in 1..nK} (ax[ix,it]*phi[ix,it]); # seventh-order polynomial
var fapp {ix in 1..nx, it in 1..nT-1} = poly[ix,it]; # function approximation of value function

param phiprime {ik in 1..nK, ix in 1..nx, it in 1..nT-1} = (ik-1)*(x[ix,it]-xbar[it])^(ik-2);
var polyprime {ix in 1..nx, it in 1..nT-1} = sum {ik in 1..nK} (ax[ix,it]*phiprime[ix,it]);
var fappprime {ix in 1..nx, it in 1..nT-1} = polyprime[ix,it]; # derivative

# time t+1
var phi1 {ik in 1..nK, ix in 1..nx, it in 2..nT-1} = (r*x[ix,it-1]-cons[ix,it-1]-xbar[it])^(ik-1);
var poly1 {ix in 1..nx, it in 2..nT-1} = sum {ik in 1..nK} (ax[ix,it]*phi1[ix,it]);
var fapp1 {ix in 1..nx, it in 2..nT-1} = poly1[ix,it]; # function approximation of next period value function
var phi1prime {ik in 1..nK, ix in 1..nx, it in 2..nT-1} = (ik-1)*(r*x[ix,it-1]-cons[ix,it-1]-xbar[it])^(ik-2);

# bellman error
var bell1 {ix in 1..nx, it in 1..nT-1} = u[ix,it] + beta*VV1[ix,it+1];
var bellerror {ix in 1..nx, it in 1..nT-1} = fapp[ix,it]-bell1[ix,it];
var lambdab {it in 1..nT-1} >= 0;

# euler error
var eulererror1 {ix in 1..nx, it in 1..nT-1} = uprime[ix,it]-beta*VVprime1[ix,it+1];
var lambdae1 {ix in 1..nx, it in 1..nT-1} >= 0;

# envelope error
var envelopeerror {ix in 1..nx, it in 1..nT-1} = polyprime[ix,it]-beta*VVprime1[ix,it+1];
var lambdaenv {it in 1..nT-1} >= 0;
AMPL code for a Simple Life-Cycle Model

minimize obj: sum {ix in 1..nx, it in 1..nT-1} (lambdae1[ix,it]+lambdab[it]+lambdaenv[it]);
subject to consterror3 {ix in 1..nx, it in 1..nT-1}: eulererror1[ix,it] >= -lambdae1[ix,it];
subject to consterror4 {ix in 1..nx, it in 1..nT-1}: eulererror1[ix,it] <= lambdae1[ix,it];
subject to consterror5 {ix in 1..nx, it in 1..nT-1}: bellerror[ix,it] >= -lambdab[it];
subject to consterror6 {ix in 1..nx, it in 1..nT-1}: bellerror[ix,it] <= lambdab[it];
subject to consterror9 {ix in 1..nx, it in 1..nT-1}: envelopeerror[ix,it] >= -lambdaenv[it];
subject to consterror10 {ix in 1..nx, it in 1..nT-1}: envelopeerror[ix,it] <= lambdaenv[it];
subject to consbound1 {ix in 1..nx, it in 1..nT-1}: r*x[ix,it]-cons[ix,it] >= xmin[it+1];
subject to consbound2 {ix in 1..nx, it in 1..nT-1}: r*x[ix,it]-cons[ix,it] <= xmax[it+1];
#last period value function and derivatives
subject to VVTconst1 {ix in 1..nx}: VV1[ix,nT] = log(1+x[ix, nT-1]-cons[ix, nT-1]);
subject to VTprime1 {ix in 1..nx}: VVprime1[ix,nT] = 1/(1+x[ix, nT-1]-cons[ix, nT-1]);
# period 2..nT-1 value function and derivatives
subject to VVTminus1 {ix in 1..nx, it in 2..nT-1}: VV1[ix, it] = fapp1[ix, it];
subject to VTminus1prime {ix in 1..nx, it in 2..nT-1}: VVprime1[ix, it] = sum {ik in 1..nK} (ax[ik, it]*phi1prime[ik,ix,it]);
data;
param xmin :=
1 1
2 0.5
3 0.01;
param xmax :=
1 10
2 10
3 10;
Continuous and Discrete State Variables

Let $W$ be a continuous state variable and $J$ be a discrete state variable. Time $t$ value function is approximated by

$$V_t(W, J) = \Phi(W, J; a_t, \bar{W}_t) = \sum_{k=0}^{7} a_{k+1,t}(W - \bar{W}_t)^k$$

The constrained optimization approach to a life-cycle model with continuous and discrete state variables is:

$$\text{Minimize} \sum_i \sum_j \sum_t \lambda^e_{i,j,t} + \sum_j \sum_t \lambda^b_{j,t} + \sum_j \sum_t \lambda^{env}_{j,t}$$

subject to
- Bellman Error:
  $$-\lambda^b_{j,t} \leq \Phi(W, J; a_t) - [u(c^*, J) + \beta \Phi(RW - c^*, J; a_{t+1})] \leq \lambda^b_{j,t}$$
- Euler Error
  $$-\lambda^e_{i,j,t} \leq u'(c^*, J) - \beta R \Phi'(RW - c^*, J; a_{t+1}) \leq \lambda^{env}_{i,j,t}$$
- Envelope Error:
  $$-\lambda^{env}_{j,t} \leq \Phi'(W, J; a_t) - \beta \Phi'(RW - c^*, J; a_{t+1}) \leq \lambda^{env}_{j,t}$$

Valentina Michelangeli  
Boston University  

Does it Pay to Get a Reverse Mortgage?
Empirical Part

- We have continuous data on wealth and consumption.
- We assume that the measurement error in consumption is normally distributed with mean 0 and unknown variance $\sigma^2$.
- We can use the Euler Equation to recover the predicted value of consumption.
- The probability that household $n$ chooses consumption $c_{n,tp}$ in period $tp$ is:

$$
Pr(c_{n,tp}| W_{n,tp}^{data}) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(c_{n,tp}^{data} - c_{n,tp}^{pred})^2}{2\sigma^2}}
$$

Therefore the Log-Likelihood is given by:

$$
\mathcal{L}(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log Pr(c_{n,tp}| W_{n,tp}^{data}, \theta)
$$
Structural Estimation with DP

- Conventional Approach
  1. take a guess of structural parameters
  2. solve DP
  3. calculate loglikelihood
  4. repeat 1,2,3 until loglikelihood is maximized

- The constrained optimization approach to structural estimation with dynamic programming is:

\[
\text{Max} \mathcal{L} - \text{Penalty} \cdot \Lambda
\]

subject to:
- Euler error
- Bellman error
- Envelope error

where \( \Lambda = \sum_t \sum_i \lambda_{i,t}^{e} + \sum_t \lambda_{t}^{b} + \sum_t \lambda_{t}^{env} \)
For most retirees the house is their major asset.

A reverse mortgage (RM) allows one to convert equity in a house property into an income stream, without making the periodic loan payments or moving.

Build a structural, dynamic model of retirees’ consumption, housing and moving decisions and solve using MPEC (Mathematical Programming with Equilibrium Constraints).

Welfare analysis shows that RM provides liquidity and longevity insurance. However, there are very high start up costs and moving becomes a risky proposition.
Reverse Mortgage

- RM are home loans that do not have to be repaid as long as the borrower lives in the house.
- But the minimum between the house value and the principal plus the cumulated interest has to be paid back if the retiree definitively moves out or dies (non-recourse loans).
- Potential Market: 30.8 million households with at least one member age 62 and older in 2006.
- Actual Market: 265,234 federally insured RM in 2007, about 1% of the 30.8 million households.
Is the Reverse Mortgage a Fair Contract?

Option Payment: Lump Sum at the Beginning

- Initial Cost for the Lender: $\overline{B}$
- Closing Cost: $F = \lambda H_{it} + f$
- Outstanding Debt: $G_{it}^{RM} = \overline{B} \sum_{j=1..t} (1 + i_D)^{t-j}$
- Repayment in Present Value: $RM_{it} = \frac{\min(H_{it}, G_{it}^{RM})}{R^{t-j}}$
- Expected Gain for the Lender:
  $EGain_{j,i} = F + \sum_{t=j+1..T} \eta_{i,t-1} [(1 - \eta_{i,t})(1 - m_{i,t}) + \eta_{i,t} m_{i,t}] RM_{i,t}$
- Closure contract at age 62, H=$100,000

Under this setting:

- Annually Adj HECM: $\overline{B}=$$31,000  EG=$64,000  EG(F=0)=$54,000
- Monthly Adj HECM: $\overline{B}=$$47,000  EG=$74,000  EG(F=0)=$63,000
- FM HomeKeeper: $\overline{B}=$$10,000  EG=$30,000  EG(F=0)=$22,000
Model

- Preferences

\[ U_{it}(C_{it}, H_{it}) = \frac{(C_{it}^{1-\omega} H_{it}^\omega)^{1-\gamma}}{1 - \gamma} + \varepsilon_{it}(d_{it}) \]

where \( \varepsilon_{it}(d_{it}) \) is a vector of unobserved utility components associated to the discrete housing choice and it is Extreme Value Type I distributed.

- Budget Constraint

\[ W_{it+1} = (1 + r) W_{it} + ss - C_{it} - \psi_{it} - M_{it} \]

- Choice set:
  - Continuous \( C_{it} \)
  - Discrete \( d_{it} \): Move-Stay \( D^M_{it} \), Own-Rent \( D^O_{it} \), House \( H_{it} \)
Housing Expenses

- Per-Period Cost

\[ \psi_{it} = [D_i^O \psi_{own} + (1 - D_i^O) \psi_{rent}][D_{it}^M H_{it+1} + (1 - D_{it}^M)H_{it}] \]

- Moving Cost

\[ M_{it} = D_{it}^M D_{it}^O [D_{it+1}^O H_{it+1} - H_{it} + H_{it+1} \phi(D_{it+1}^O)] + D_{it}^M (1 - D_{it}^O) H_{it+1} \phi_{rent} \]

where the transaction costs are:

\[ \phi(D_{it+1}^O) = [D_{it+1}^O \phi_{own} + (1 - D_{it+1}^O) \phi_{rent}] \]
Value Function

\[ V_{it}(X_{it}, \varepsilon_{it}) = \max_{d_{it}, C_{it}} \frac{(C_{it}^{1-\omega}(D_{it}^{M}H_{it+1}^{\omega} + (1-D_{it}^{M})H_{it}^{\omega}))^{1-\gamma}}{1-\gamma} + \varepsilon_{it}(d_{it}) + \beta(n_{t+1}E[V_{it}(W_{it+1}, (D_{it}^{M}H_{it+1} + (1-D_{it}^{M})H_{it}), D_{it+1}^{O}, \varepsilon_{it+1})] + b(TW_{it+1})) \]

subject to

\[ W_{it+1} = (1 + r)W_{it} + ss - C_{it} - \psi_{it} - M_{it} \]

\[ C_{it} \geq C_{MIN} \]

State Space

\[ X_{it} = \{W_{it}, H_{it}, D_{it}^{O}, Age_{it}\} \]

Preference parameters to estimate

\[ \theta = \{\gamma, \omega, \sigma\} \]
Assumption I: Additivity

\[ U(d_{it}, C_{it}, X_{it}, \theta) = \frac{(C_{it}^{1\omega} H_{it}^\omega)^{1-\gamma}}{1-\gamma} + \varepsilon_{it}(d_{it}) \]

Assumption II: Conditional Independence

\[ f(X_{it+1}, \varepsilon_{it+1}|d_{it}, C_{it}, X_{it}, \varepsilon_{it}, \theta) = q(\varepsilon_{it+1}|X_{it+1})g(X_{it+1}|d_{it}, C_{it}, X_{it}, \theta) \]

Bellman Equation:

\[ V_t(X_{it}, \varepsilon_{it}) = \max_{d_{it}, C_{it}} [U(d_{it}, C_{it}, X_{it}, \theta) + \varepsilon_{it}(d_{it}) + \beta \eta_{it+1} EV(X_{it+1})] \]

\[ = \max_{d_{it}} \left\{ \max_{C_{it}} \left\{ [U(d_{it}, C_{it}, X_{it}, \theta) + \beta \eta_{it+1} EV(X_{it+1})]|d_{it} \right\} + \varepsilon_{it}(d_{it}) \right\} \]
Inner Maximization (consumption conditional on housing)

\[ r(X_{it}, d_{it}) = \max_{C_{it}} [U(d_{it}, C_{it}, X_{it}, \theta) + \beta \eta_{it+1} EV_{t+1}(X_{it+1})] | d_{it} ] \]

Outer Maximization (housing)
Conditional Choice Probabilities

\[ P(j | X_{it}, \theta) = \frac{\exp \{ r(X_{it}, j, \theta) \}}{\sum_{k \in d_{it}(X_{it})} \exp \{ r(X_{it}, k, \theta) \}} \]

where

\[ EV_{t+1}(X_{it+1}) = \ln \left[ \sum_{k \in d_{t}(X_{t})} \exp \{ r(X_{it}, k, \theta) \} \right] \]
Dynamic Programming

- Minimize the Sum of Errors

\[ \Lambda = \sum_t \sum_i \sum_j \sum_Q \lambda^e_{i,j,Q,t} + \sum_t \sum_j \sum_Q \lambda^b_{j,Q,t} + \sum_t \sum_j \sum_Q \lambda^{env}_{j,Q,t} + \sum_t \sum_i \sum_j \sum_Q \sum_d \lambda^{cons}_{i,j,d,Q,t} \]

subject to:

\[ -\lambda^e_{i,j,Q,t} \leq EulerEquation_{i,j,Q,t} \leq \lambda^e_{i,j,Q,t} \]
\[ -\lambda^b_{j,Q,t} \leq BellmanEquation_{j,Q,t} \leq \lambda^b_{j,Q,t} \]
\[ -\lambda^{env}_{j,Q,t} \leq EnvelopeCondition_{j,Q,t} \leq \lambda^{env}_{j,Q,t} \]
\[ -\lambda^{cons}_{i,j,d,Q,t} \leq PolicyFunction_{i,j,d,Q,t} \leq \lambda^{cons}_{i,j,d,Q,t} \]
Solving DP and Estimation with the MPEC

\[
\max_{\theta, a, c} \mathcal{L}(\theta) - \text{Penalty} \cdot \Lambda
\]

subject to:
Euler errors
Bellman error
Envelope error
Policy function error

where

\[
\mathcal{L}(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log \Pr(d_{n,tp}^H, c_{n,tp} | W_{n,tp}^{data}, H_{n,tp}^{data}, \theta)
\]

We assume that there is a measurement error in consumption \( \sim N(0, \sigma^2) \)
Data

The Health and Retirement Study (HRS) and The Consumption and Activities Mail Survey (CAMS). US data (2000-2005)

- We select a group of 175 households that could be the potential target segment for Reverse Mortgage
- Characteristics: 62 years old or older, single, retiree, homeowner, complete information about consumption and financial situation

<table>
<thead>
<tr>
<th></th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>$40,000</td>
<td>$70,000</td>
<td>$92,000</td>
</tr>
<tr>
<td>W</td>
<td>$5,000</td>
<td>$17,500</td>
<td>$63,000</td>
</tr>
<tr>
<td>H/W</td>
<td>0.86</td>
<td>2.5</td>
<td>7.5</td>
</tr>
<tr>
<td>C</td>
<td>$6,270</td>
<td>$9,774</td>
<td>$15,090</td>
</tr>
<tr>
<td>ss</td>
<td>$6,972</td>
<td>$9,468</td>
<td>$11,340</td>
</tr>
<tr>
<td>Age</td>
<td>69</td>
<td>74</td>
<td>79</td>
</tr>
</tbody>
</table>
Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Asymptotic s.e.</th>
<th>Bootstrap s.e</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>3.87</td>
<td>(1.07e – 009)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.85</td>
<td>(1.82e – 009)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.87</td>
<td>(6.82e – 004)</td>
<td>(0.05)</td>
</tr>
</tbody>
</table>

- The asymptotic standard errors, computed using a finite difference approach, show a big small-sample bias.
- We use a bootstrap procedure to reduce the small-sample bias.
Simulation of Welfare Gain from Reverse Mortgage

The welfare gain from a reverse mortgage has been calculated as a percentage increase in the initial non-housing financial wealth that makes the household without reverse mortgage as well off in expected utility terms as with the reverse mortgage.

- Welfare Gain as a Function of the House Value

<table>
<thead>
<tr>
<th>House Value</th>
<th>Financial Wealth</th>
<th>Welfare Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$40,000</td>
<td>$10,300</td>
<td>-37.95%</td>
</tr>
<tr>
<td>$80,000</td>
<td>$26,000</td>
<td>3.16%</td>
</tr>
<tr>
<td>$120,000</td>
<td>$48,800</td>
<td>25.56%</td>
</tr>
</tbody>
</table>
Simulation of Welfare Gain from Reverse Mortgage

- Welfare Gain as a Function of the Initial Financial Wealth

<table>
<thead>
<tr>
<th>Financial Wealth</th>
<th>Median FW</th>
<th>Welfare Gain</th>
<th>House Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LW ($&lt;10,000)</td>
<td>$1,540</td>
<td>-63.7%</td>
<td>$40,000</td>
</tr>
<tr>
<td>MW ($10,000 - $80,000)</td>
<td>$26,500</td>
<td>13.44%</td>
<td>$80,000</td>
</tr>
<tr>
<td>HW ($&gt;80,000)</td>
<td>$138,000</td>
<td>34.61%</td>
<td>$80,000</td>
</tr>
</tbody>
</table>

- PROS of RM: liquidity and longevity insurance
- CONS of RM: high closing cost and moving risk
- In the moving case, households with low financial wealth are significantly worse off. Closing a RM contract dramatically affects any future consumption and housing decision.
Simulation of Welfare Gain from Renting

- The simulation shows that some households would be better off from a reverse mortgage.
- After about 20 years from its first appearance, the reverse mortgage market is still at 1% of its potentiality.
- What would be the welfare gain if the household chooses to encash the savings locked in the house by moving out and renting the same size house?

<table>
<thead>
<tr>
<th>House Value</th>
<th>Financial Wealth</th>
<th>Welfare Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$40,000</td>
<td>$10,300</td>
<td>498%</td>
</tr>
<tr>
<td>$80,000</td>
<td>$26,000</td>
<td>366%</td>
</tr>
<tr>
<td>$120,000</td>
<td>$48,800</td>
<td>205%</td>
</tr>
</tbody>
</table>
## Simulation of Welfare Gain from Renting

<table>
<thead>
<tr>
<th>Financial Wealth</th>
<th>Median FW</th>
<th>Welfare Gain</th>
<th>House Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LW ( &lt; $10,000)</td>
<td>$1,540</td>
<td>5200%</td>
<td>$40,000</td>
</tr>
<tr>
<td>MW ($10,000 - $80,000)</td>
<td>$26,500</td>
<td>215%</td>
<td>$80,000</td>
</tr>
<tr>
<td>HW ( &gt; $80,000)</td>
<td>$138,000</td>
<td>23%</td>
<td>$80,000</td>
</tr>
</tbody>
</table>

- Households with very low financial wealth have a significant welfare loss from closing a reverse mortgage contract, due to the moving risk, the high transaction costs and the fact that they could borrow only a percentage of the house value.

- On the other side, if the equity in their house is released by moving out and renting, they would be able to en-cash the full amount of the saving locked in their house without increasing their level of indebtedness and without incurring the moving risk.
Conclusions

- Innovative structural, dynamic model of retirees’ consumption, housing and moving decisions.

- First MPEC approach applied to an empirical structural model with dynamic programming problem and continuous state variables.

- Reverse mortgage provides liquidity and longevity insurance. However moving becomes a much riskier proposition, especially for households with low financial wealth.

- Common belief is that Reverse mortgage is for “house rich but cash poor” households. This paper shows otherwise.

Thank you
Appendix: DP with Approximation of the Value Function

- Euler Equations:
  \[ u'(c^*_{dN}, H) - \beta \eta_{t+1} R V'_{t+1}(RW - c^*_{dN} - \psi + ss; H, Q) = 0 \]
  \[ u'(c^*_{dMhq}, h) - \beta \eta_{t+1} R V'_{t+1}(RW - c^*_{dMhq} - \psi - M + ss; h, q) = 0 \]

- Bellman Equation:
  \[ V_t(W, H, Q) = \ln \left\{ \exp(\hat{V}_{dN,t}) + \sum_q \sum_h \exp(\hat{V}_{dMhq,t}) \right\} \]

- Envelope Condition:
  \[ V'_t(W, H, Q) = \Pr(NM|W, H, Q) \cdot \hat{V}'_{dN,t} + \sum_q \sum_h \Pr(Mhq|W, H, Q) \cdot \hat{V}'_{dMhq,t} \]
Value Function Approximation

\[ V_t(W, H, Q) = \Phi(W, H, Q; a_t, \overline{W}_t) = \sum_{k=0}^{7} a_{k+1,H,Q,t}(W - \overline{W}_t)^k \]

Policy Function Approximation

\[ c_{d,t}^*(W, H, Q) = \Phi(W, H, Q; b_{d,t}, \overline{W}_t) = \sum_{k=0}^{7} b_{k+1,H,Q,d,t}(W - \overline{W}_t)^k \]

We would like to find coefficients \( a_t \) and \( b_{d,t} \) such that each time \( t \) Bellman equation, along with the Euler and Envelope conditions, holds with the \( \Phi \) approximation.
Euler Errors

\[-\lambda^e_{i,j,Q,t} \leq u'(c^*_{i,j,dN,t}, H_j,t) - \beta R\Phi'(RW_{i,t} - c^*_{i,j,dN,t} - \psi + ss; H_j,t, Q_t; a_{t+1}) \leq \lambda^e_{i,j,Q,t}\]

\[-\lambda^e_{i,j,Q,t} \leq u'(c^*_{i,j,dMhq,t}, H_{t+1}) - \beta R\Phi'(RW_{i,t} - c^*_{i,j,dMhq,t} - \psi - M + ss; H_{t+1}, Q_{t+1}; a_{t+1}) \leq \lambda^e_{i,j,Q,t}\]

Bellman Error

\[-\lambda^b_{j,Q,t} \leq \Phi(W_{i,t}, H_j,t, Q_t; a_t) - \ln \left\{ \exp(\hat{V}_{i,j,dN,t}) + \sum_q \sum_h \exp(\hat{V}_{i,j,dMhq,t}) \right\} \leq \lambda^b_{j,Q,t}\]

where

\[\hat{V}_{i,j,dN,t} = u(c^*_{i,j,dN,t}, H_j,t) + \beta \eta_{t+1} \Phi(RW_{i,t} - c^*_{i,j,dN,t} - \psi + ss; H_j,t, Q_t; a_{t+1})\]

\[\hat{V}_{i,j,dMhq,t} = u(c^*_{i,j,dMhq,t}, H_{t+1}) + \beta \eta_{t+1} \Phi(RW - c^*_{i,j,dMhq,t} - \psi - M + ss; H_{t+1}, Q_{t+1}; a_{t+1})\]
Envelope Error

\[-\lambda_{j,Q,t}^{env} \leq \Phi'(W_i,t, H_j,t, Q_t; a_t) - \{f_{i,j,dN_t} \cdot \Phi'(RW_i,t - c_{i,j,dN_t}^* - \psi + ss; H_j,t, Q_t; a_{t+1})

+ \sum_q \sum_h [f_{i,j,dMhq,t} \cdot \Phi'(RW_i,t - c_{i,j,dMhq,t} - \psi - M; H_{t+1}, Q_{t+1}; a_{t+1})] \} \leq \lambda_{j,Q,t}^{env}\]

where

\[f_{i,j,d,t} = \Pr(d | W_i,t, H_j,t, Q_t) = \frac{\exp(\tilde{V}_{i,j,d,t})}{\exp(\tilde{V}_{i,j,dN,t}) + \sum_q \sum_h \exp(\tilde{V}_{i,j,dMhq,t})}\]

Policy Function Error

\[-\lambda_{i,j,Q,d,t}^{cons} \leq \Phi(W_i,t, H_j,t, Q_t; b_t) - c_{i,j,d,t}^*(W_i,t, H_j,t, Q_t) \leq \lambda_{i,j,Q,d,t}^{cons}\]
Loglikelihood

- Measurement Error in Consumption

\[ \Pr(c_n, t | d_{n, tp}^H, W_{n, tp}^{data}, H_{n, tp}^{data}, Q_{n, tp}^{data}) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(c_{n, tp}^{data} - c_{n, tp}^{pred})^2}{2\sigma^2}} \]

- Discrete Choice Probability

\[ \Pr(d_{n, tp}^H | W_{n, tp}^{data}, H_{n, tp}^{data}, Q_{n, tp}^{data}) = \frac{e^{V_{d, n, tp}}}{\sum_m e^{V_{m, n, tp}}} \]

- Joint Probability of Housing and Consumption Choice

\[ \Pr(d_{n, tp}^H, c_n, t | W_{n, tp}^{data}, H_{n, tp}^{data}, Q_{n, tp}^{data}) = \Pr(d_{n, tp}^H | W_{n, tp}^{data}, H_{n, tp}^{data}, Q_{n, tp}^{data}) \cdot \Pr(c_n, t | d_{n, tp}^H, W_{n, tp}^{data}, H_{n, tp}^{data}, Q_{n, tp}^{data}) \]

- Log-Likelihood

\[ \mathcal{L}(\theta) = \sum_{n=1}^{N} \sum_{tp=1}^{TP} \log \Pr(d_{n, tp}^H, c_n, t | W_{n, tp}^{data}, H_{n, tp}^{data}, Q_{n, tp}^{data}, \theta) \]

Does it Pay to Get a Reverse Mortgage?