Risk, Diversification and Growth
ICE08 Project

Martí Mestieri
Tarik Ocaktan
Thom Sampson
David Wiczer

August 7, 2008
Outline

The Model

Results
The Model

- Acemoğlu and Zilibotti (JPE 1997)
- OLG economy: each generation lives for two periods.
- No population growth.
- Production sector consists of two sectors.
Production sector consists of two sectors:

- **Final goods sector** with Cobb-Douglas production function
  \[
  Y(t) = K(t)^\alpha L(t)^{1-\alpha}
  \]
  with full capital depreciation \( \delta = 1 \).

- **Intermediate sector** transforms savings \( s(t-1) \) into capital \( k(t) \) to be used for production at time \( t \). Sector consists of a continuum \([0, 1]\) of intermediates, and stochastic elements only affect this sector. There is a riskless asset \( X(t) \).
Intermediate sector:

- Possible states of nature are also within the unit interval. Intermediate sector \( j \in [0, 1] \) pays a positive return only in state \( j \) and nothing in any other state.

- Each sector has a minimum size requirement \( M(j) \) such that there are positive returns only if aggregate investment, \( I(j, t) \), in sector exceeds \( M(j) \)

\[
M(j) = \max \left\{ 0, \frac{D}{1 - \gamma} (j - \gamma) \right\} \tag{2}
\]

- Intermediate sectors \( j \leq \gamma \) have no minimum size requirement.
The Model
Production Sector

Figure: Minimum size requirements, $M(j)$, of different sectors and demand for assets, $I^*(n)$
The Model
Household Sector

- Preference of household from a generation born at time $t$

$$E_t U(c_1(t), c_2(t + 1)) = U(c_1(t)) + \beta \int_0^1 U(c_2(j, t + 1)) \, dj$$

- Each household has 1 unit of labor when young and no labor endowment when old
The Model
Timing of Events

Figure: Life cycle of a typical household
In state $j$, the aggregate stock of capital is

$$K(j, t + 1) = \int_{h \in H_t} (qX^h(t) + Ql^h(j, t))\,dh$$

- $l^h(j, t)$: amount of savings invested by young agent $h \in H_t$ in sector $j$ at time $t$
- $X^h(t)$: amount invested in the safe intermediate sector
The Model
Equilibrium Factor Prices

- **Wage equation**

\[
w(j, t + 1) = (1 - \alpha)K(j, t + 1)^\alpha
= (1 - \alpha)\left( \int_{h \in \mathcal{H}_t} (qX^h(t) + QI^h(j, t)) \, dh \right)^\alpha
\]

- **Return to investment**

\[
R(j, t + 1) = \alpha K(j, t + 1)^{\alpha - 1}
= \alpha \left( \int_{h \in \mathcal{H}_t} (qX^h(t) + QI^h(j, t)) \, dh \right)^{\alpha - 1}
\]
Households take prices and the set of available securities at time $t$ as given. The problem of the representative household $h \in \mathcal{H}_t$ is given by

$$\max_{s(t), X(t), [l(t)]_{0 \leq j \leq 1}} \left\{ U(c(t)) + \beta \int_0^1 U(c(j, t+1)) \, dj \right\}$$

s.t.

$$X(t) + \int_0^1 l(j, t) \, dj = s(t)$$

$$c(j, t + 1) = R(j, t + 1)(qX(t) + Ql(j, t))$$

$$l(j, t) = 0, \quad \forall j \not\in J(t)$$

$$c(t) + s(t) \leq w(t)$$
Outline

The Model

Results
Method
Different Approaches to Solve the Problem

- Maximization problem by stating value function and market clearing constraints.
- First order condition: useful to characterize interior solutions.
- Complementarity conditions: to try complete characterization but PATHAMPL lacks appropriate algorithm.
Results
Policy Functions

consumption functions

- young consumption
- old consumption: bad state
- old consumption: good state

percent of steady state capital stock

consumption

0 20 40 60 80 100 120
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
Results

Policy Functions

n(k) Chebyshev approximation

prob of return vs. percent of steady state capital stock
## Results

### Chebyshev Coefficients

<table>
<thead>
<tr>
<th>n</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.54481</td>
</tr>
<tr>
<td>2</td>
<td>0.02682</td>
</tr>
<tr>
<td>3</td>
<td>-0.00581</td>
</tr>
<tr>
<td>4</td>
<td>0.00266</td>
</tr>
<tr>
<td>5</td>
<td>-0.00152</td>
</tr>
<tr>
<td>6</td>
<td>0.00098</td>
</tr>
<tr>
<td>7</td>
<td>-0.00068</td>
</tr>
<tr>
<td>8</td>
<td>0.00049</td>
</tr>
<tr>
<td>9</td>
<td>-0.00037</td>
</tr>
<tr>
<td>10</td>
<td>0.00028</td>
</tr>
<tr>
<td>11</td>
<td>-0.00022</td>
</tr>
<tr>
<td>12</td>
<td>0.00017</td>
</tr>
<tr>
<td>13</td>
<td>-0.00014</td>
</tr>
<tr>
<td>14</td>
<td>0.00011</td>
</tr>
<tr>
<td>15</td>
<td>-0.00009</td>
</tr>
<tr>
<td>16</td>
<td>0.00007</td>
</tr>
<tr>
<td>17</td>
<td>-0.00005</td>
</tr>
<tr>
<td>18</td>
<td>0.00003</td>
</tr>
<tr>
<td>19</td>
<td>-0.00002</td>
</tr>
<tr>
<td>20</td>
<td>0.00001</td>
</tr>
</tbody>
</table>
Results

Chebyshev Approximation Residuals

residuals on the young consumption function

residual: approximation − linear

percent of steady state capital stock

residual: approximation − linear
Results

Approximation Accuracy

\[ \text{FOC: } c_y^{-\sigma} = R_{\text{good}} \cdot Q \cdot c_{\text{good}}^{-\sigma} \]

percent of steady state capital stock
Results

Safe Investment

safe investment / consumption
Results

Chebyshev Approximation at the Kink

Missing the kink: $n(k) \rightarrow 1$
Results

Chebyshev Approximation at the Kink

Safe Investment Decision

Percent of Steady State Capital
Results

Simulated Trajectory
### Data
Sample: 1970-2006

<table>
<thead>
<tr>
<th>Year 1970</th>
<th>(*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per capita GDP mean for rich countries (A)</td>
<td>13 863</td>
</tr>
<tr>
<td>Per capita GDP mean for emerging countries (B)</td>
<td>2 075</td>
</tr>
<tr>
<td>Ratio (A/B)</td>
<td>6.68</td>
</tr>
<tr>
<td>Ratio of capital</td>
<td>225</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample 1970-2006</th>
<th>(*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean GDP growth rate in rich countries</td>
<td>2.24%</td>
</tr>
<tr>
<td>Mean GDP growth rate in emerging countries</td>
<td>2.22%</td>
</tr>
<tr>
<td>Average variance in rich countries</td>
<td>0.05%</td>
</tr>
<tr>
<td>Average variance in emerging countries</td>
<td>0.17%</td>
</tr>
</tbody>
</table>

(*) GDP per capita (constant 2000 USD)
Data
Simulating Moments

- Back of the envelope calculation using a Cobb-Douglas production function $y = k^\alpha 1^{1-\alpha}$ function with parameter $\alpha = 0.35$, implies that an average developing country has only a .5% stock of capital of the richest.

- Fixing the coefficient or risk aversion to 4 and the share of risky activities without no fix cost in the economy to 10%. Fixing $q = 1$, given that what matters is the relative payment $Q/q$, we have have simulated the moments that our model predicts for different levels of $D$ and $Q$.

- We have tried to match two moments: the average growth and variance of emerging countries relative to the developed. We have 2 free variables to match 2 moments. The solution seems to be around $D = .85, Q = 5.5$. 