Introduction to Optimization and Generalized Equations

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Optimization is Fundamental in Economics Modelling

- What is economics:
  - Definition: The study of the allocation of scarce resources
  - Assumption: actors make choices that maximize an objective function
  - Hence, economics problems are constrained optimization problems: maximize objective subject to scarcity constraints
- Examples
  - Consumer choice
  - Social planning problems
  - Principal-agent problems
  - Life-cycle problems
  - Profit maximization
  - Portfolio choice

Equations are Fundamental in Economics Modelling

- Equilibrium: a collection of choices by economic actors that are consistent with scarcity and individual rationality
- Demand equals supply
  - Competitive equilibrium
  - Asset market equilibrium
  - Dynamic market equilibrium
- Nash-Cournot
  - Oligopoly theory
  - Games of incomplete information
  - Games of asymmetric information
  - Political games

All Economic Analysis Uses Optimization and Equations

- Analysis of economic data is an optimization problem
  - Unknown parameters are chosen so as to maximize the compatibility between statistical model and data
    - Least squares methods
    - Method of moments
    - Maximum likelihood
  - Unknown parameters are chosen to fit data and satisfy equilibrium conditions
    - Structural estimation
    - A constrained optimization problem

Numerical Analysis is an Example of an Economics Problem

- Numerical analysis is the development of computational tools that best use scarce computational resources to accomplish a computational task
- Scarce resources
  - Computer time
  - Programmer time
  - Progammer ability
- Objective
  - Accuracy
  - Speed
- Technologies
  - Memory
  - Processor
  - Communication links

- Computation is about using *computers*
  - You need to understand what computers do
    - Numbers - stored with finite precision
    - Operations - executed with small errors
    - Storage methods and cache management
    - Interpreted versus compiled code
  - As computer technologies change, the choice of algorithms changes
    - Single precision to double precision
    - Expensive memory to cheap memory
    - Serial to parallel processing

Conventional "Wisdom" versus ICE09

Optimization Methods

- CW: There has been no advances in optimization algorithms in the past 45 years that are useful in economics
- ICE09: Todd Munson, winner of a Presidential Early Career Award for Scientists and Engineers in 2006, will survey the advances beginning with Robert Wilson's 1964 Ph. D. thesis and continuing through today.

Optimization Software

- CW: Stay with simple methods, motivated by economic intuition; stay away from "magical black boxes"
- ICE09: A box ceases to be black when you open your eyes and turn on the lights, which we will help you do in our software tutorial sessions.

Estimation

- CW: One has to sacrifice efficiency and limit yourself to "computationally light" estimators in order to compute a consistent estimator
- ICE09: Prof. Che-Lin Su will show you how to use modern optimization methods to compute efficient estimators

Estimation

- CW: Nelder-Mead is all you need in empirical IO
- ICE09: You will learn that it is appropriate to give Nelder-Mead code the name "Amoeba", a one-celled animal with no nervous system.

Dynamic Programming

- CW: It is difficult to write DP code that is stable, efficient, and accurate, particularly for multidimensional problems.
- ICE09: It is easy to do this for concave problems; example: dynamic portfolio allocation of several assets (stocks, a bond, and options) with proportional transaction costs.

Bayesian Methods

- CW: Bayesian methods are intractable due to high-dimensional integrals
- ICE09: Peter Rossi will show you efficient methods for Bayesian methods and also show you software you can use.

Dynamic Games

- CW: Finding feedback equilibria (a.k.a. MPE) is slow and difficult.
- ICE09: Life is always hard if you use Gauss-Jacobi methods. Karl Schmedders will show you how to use complementarity methods to solve dynamic games, even ones where players occasionally hit constraints such as zero output

Computer Architecture

- CW: It is difficult to use parallel systems.
- ICE09: Steve Wright and Greg Thain will demonstrate Condor, a distributed computing system developed at the University of Wisconsin.

Polynomial Systems

- CW: There are no closed-form solutions for polynomial systems of equation
- ICE09: Karl Schmedders and Felix Kubler will demonstrate tools from algebraic geometry that allow you to solve polynomial systems, and prove theorems about economic models.

Numerical Integration

- CW: It is not tractable to accurately compute integrals of the form

$$\int_{x_1}^{\infty}\int_{x_2}^{\infty}\ldots\int_{x_n}^{\infty} e^{-x^\top \Sigma^{-1} x}\, dx$$

- ICE09: It helps to know some math and have good software.

Dynamic Supergames

- CW: It is difficult to solve interesting dynamic supergames.
- ICE09: Sevin Yeltekin will demonstrate new algorithms for solving supergames with two and three players, and many states, and use them to find all Nash equilibria of dynamic investment games.

Value of Knowledge

- CW: "Yes, we cannot trust his results because of his computational errors, but he can easily learn about computation after he finishes his thesis and starts his job"
- ICE09: It takes work to learn serious computational methods and acquire useful computational skills, but the result is worth the effort.