KNITRO for Nonlinear Optimization

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Outline

- Overview of the KNITRO product
- KNITRO 5.0 features
- Questions
- Examples
  - Strategic bidding with MPECs
  - Sample code

KNITRO Overview

KNITRO Product

- Solve mathematical optimization problems
- **Robust** (3 different solvers)
- **Large-scale** (sparse linear algebra)
- **Embeddable** (multiple platforms, thread-safe)
- **Commercially supported** (Ziena since 2001)

Mathematical Problem Form

- KNITRO solves problems that can be written as

$$\begin{aligned} \min_{x \in \mathbf{R}^n} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \geq 0 \quad i \in I \\ & h_j(x) = 0 \quad j \in E \\ & l_i \leq x_i \leq u_i \end{aligned}$$

- *x* must be continuous
- *f(x)*, *g(x)*, *h(x)* must be smooth
- No convexity requirement

Optimization Problem Types

- General nonlinear problems with constraints
- Unconstrained
- Bound constrained
- Equality constrained
- Complementarity constrained (MPECs)
- Nonlinear systems of equations
- Least squares (linear and nonlinear)
- Linear programs (LPs)
- Quadratic programs (convex and nonconvex)

Where is KNITRO Used?

Optimal power flow

Strategic bidding, option pricing

Portfolio optimization, financial investments

Semiconductor circuit layout

Chemical process control

Engineering design

Academic research (many applications)

KNITRO is Flexible Software

- Full platform support for
  - Windows XP/2003/2000 (Windows x64)
  - Linux: 32-bit and 64-bit
  - Mac OS X (PowerPC, x86)
  - Solaris (SPARC, x86)
- Simple API for applications coded in
  - C, C++, Fortran, Java
- Available as a solver for
  - AMPL
  - Frontline (Excel)
  - GAMS
  - Mathematica
  - Matlab (Tomlab)

KNITRO is Commercial Software

- Embeddable libraries
  - Examples provided in C, Fortran, Java
  - Thread-safe
  - Debugging capabilities
- Ziena incorporated in 2001
- Ziena license manager
- Ziena support (Annual Maintenance Service)
  - Software assistance, bug fixes
  - Optimization problem formulation
  - Free upgrades for new features

KNITRO Availability

- **http://www.ziena.com**
- **Free** “student” download
  - 300 variable limit
- **Free** 30-day trial of full strength version
  - **Ask for a CD**!
- Academic pricing for University research

KNITRO 5.0 Features

KNITRO Release History

5.0 – Currently Available

- 5.0.1 (Apr 2006) – bug fix
- 5.0.0 (Feb 2006) – major release

4.0 – No longer available

- 4.0.7 (Jan 2006) – last bug fix
- 4.0.0 (Oct 2004) – major release

3.1 – No longer available

- 3.1.0 (Sep 2003) – minor release
- 3.0.0 (Apr 2003) – major release

New in KNITRO 5.0

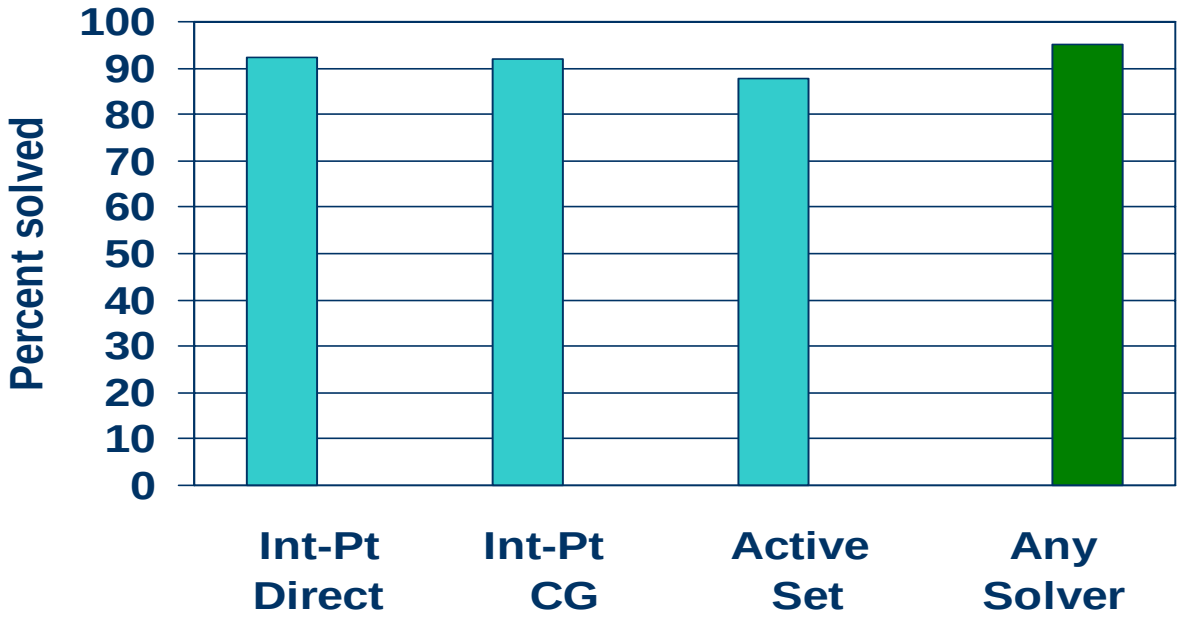
- Capability to solve **MPEC** problems
- **Multi-start** option for trying to find the global solution
- **Cross over** from an interior-point solution to the Active Set solver for highly accurate solutions
- Additional barrier update rule
- Streamlined programming interface
- Built-in derivative checking
- Improvements in algorithm efficiency and robustness

3 Different Solvers/Algorithms

- Interior-point / barrier
  - KNITRO Interior/Direct (**alg=1**)
    (handles ill-conditioned problems)
  - KNITRO Interior/CG (**alg=2**)
    (handles large/dense Hessians)
- Active Set SLQP
  - KNITRO Active (**alg=3**)
    (able to warm start, cross over from Interior-point solvers)
- All state-of-the-art, supported by ongoing academic research, proven globally convergent

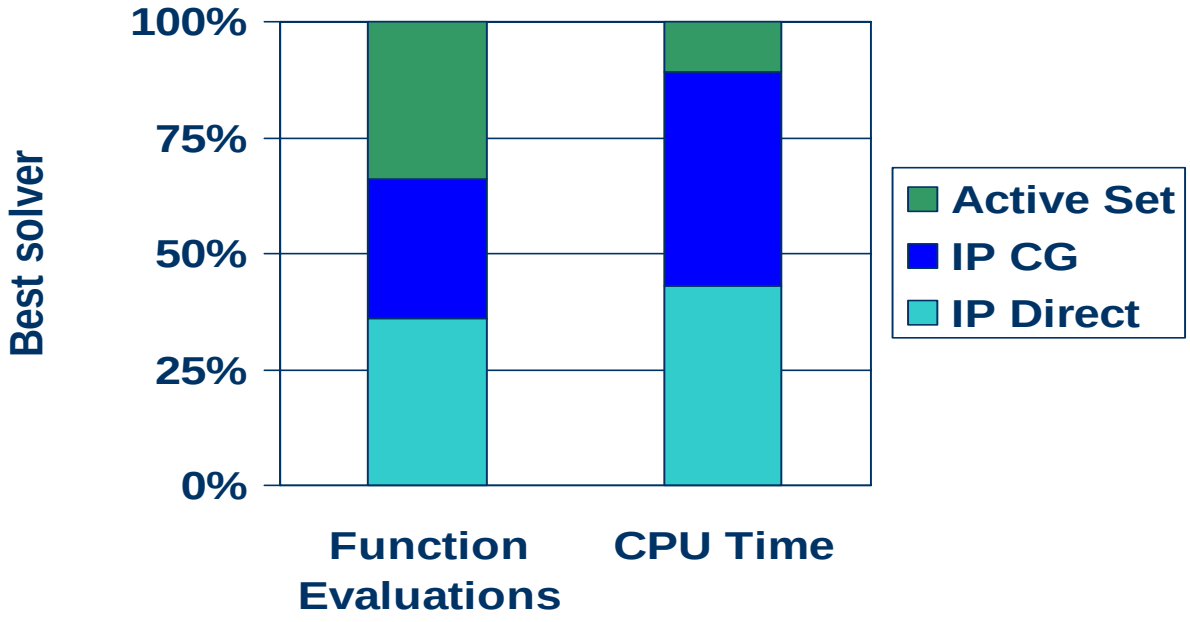
3 Solvers are more Robust

- 920 test problems
- 95.2% can be solved by at least one solver



3 Solvers are more Efficient

- 920 test problems
- Compare solvers when both find a solution



MPEC Problems

Math **P**rogramming with **E**quilibrium **C**onstraints
aka: Complementarity Constraints / MPCC / NCP

$$\min f(x)$$
$$\text{s.t. } g(x) \geq 0$$
$$h(x) = 0$$
$$\boxed{0 \leq x_1 \perp x_2 \geq 0}$$

Applications:
- Strategic bidding
- Economic models
- Contact problems
- Traffic equilibrium
- Disjunctive conditions

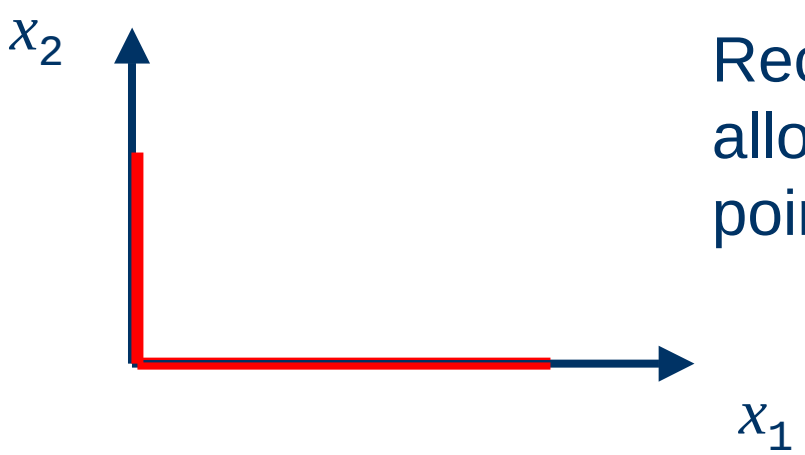
The complementarity condition means

$x_1 \geq 0, x_2 \geq 0$ and

either $x_1 = 0$ or $x_2 = 0$

Why is MPEC Hard?

$$\min f(x)$$
$$\text{s.t. } g(x) \geq 0$$
$$h(x) = 0$$
$$\boxed{0 \leq x_1 \perp x_2 \geq 0}$$



- Constraint qualification fails at feasible points
- Multipliers undefined, no interior path

Recent research breakthroughs allow reformulation for interior-point solvers

Cross Over

- Cross over cleans up the interior-point solution by switching to the Active Set solver at the end
- Cleanly identifies active set of constraints
- More accurate solution and sensitivities
- Standard for LP but very difficult for NLP
- Optional in KNITRO - may be some overhead cost

Questions ?

Examples

Strategic Bidding for Electricity

- Your company sells electric power
- You and other producers submit competitive bids to generate power
- An ISO purchases at a single “spot price”

- Your most profitable strategy is:
  - submit bids $\leq$ spot price,
  - submit bids to drive up the spot price

Strategic Bidding for Electricity

$b_{i,j}$ bid price by company i, plant j
$c_{i,j}$ generating cost per kilowatt
$G_{i,j}$ generating capacity
$g_{i,j}$ kilowatts to generate for ISO

ISO problem:

$$\begin{aligned} \min_{g_{i,j}} \quad & \sum_{i,j} b_{i,j} g_{i,j} \\ \text{subject to} \quad & 0 \leq g_{i,j} \leq G_{i,j} \\ & \sum_{i,j} g_{i,j} \geq \text{Demand} \end{aligned}$$

Spot price π_D is multiplier

Strategic Bidding for Electricity

Your problem ($i = 1$):

$$\max_{b_j} \quad \sum_j (\pi_D - c_j) g_j$$

$$\text{subject to} \quad \begin{cases} c_j \leq b_j \\ \text{ISO solution} \end{cases}$$

ISO problem: $b_{i,j} \rightarrow g_{i,j}, \pi_D$

- Bilevel linear optimization problem
- Can solve iteratively, but converges slowly

Strategic Bidding for Electricity

- Replace ISO problem with KKT conditions
- Solve as a nonlinear MPEC

$$
\begin{aligned}
b_{i,j} - \pi_D + \pi^U_{i,j} - \pi^L_{i,j} = 0 \quad & \forall i,j \\
0 \le g_{i,j} \le G_{i,j} \quad & \forall i,j \\
\sum_{i,j} g_{i,j} \ge Demand \quad &
\end{aligned}
$$

$$
\begin{aligned}
0 \le \pi_D \ \perp \ \sum_{i,j} g_{i,j} - Demand \ge 0 \\
0 \le \pi^U \ \perp \ G - g \ge 0 \\
0 \le \pi^L \ \perp \ g \ge 0
\end{aligned}
$$

Strategic Bidding for Electricity

- KNITRO solves the reformulated MPEC
- What about bids from competitors?
- Use stochastic optimization
  - Same nonlinear MPEC formulation
  - Larger, but KNITRO is for large-scale problems

Sample Code for AMPL interface

```
# testproblem.mod
# Example AMPL model to demonstrate using KNITRO with AMPL.

# Define variables and enforce that they be non-negative.
var x{j in 1..3} >= 0;

# Objective function to be minimized.
minimize obj:
        1000 - x[1]^2 - 2*x[2]^2 - x[3]^2 - x[1]*x[2] - x[1]*x[3];

# Equality constraint.
s.t. c1: 8*x[1] + 14*x[2] + 7*x[3] - 56 = 0;

# Inequality constraint.
s.t. c2: x[1]^2 + x[2]^2 + x[3]^2 -25 >= 0;

# Define initial point.
data;
let x[1] := 2;
let x[2] := 2;
let x[3] := 2;
```

Sample Code for AMPL interface

```
ampl: option solver knitro-ampl;
ampl: option knitro_options "outlev=0 alg=2";
ampl: model testproblem.mod;
ampl: solve;

KNITRO 5.0:
outlev=0
alg=2
KNITRO 5.0: LOCALLY OPTIMAL SOLUTION FOUND.
objective 9.360004e+02; feasibility error 0.000000e+00
6 major iterations; 7 function evaluations

ampl:
```

Thanks