Optimization and Equations:
Connections Between Economics and Numerical Analysis

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Optimization is Fundamental in Economics Modelling

- What is economics:
  - Definition: The study of the allocation of scarce resources
  - Assumption: actors make choices that maximize an objective function
  - Hence, economics problems are constrained optimization problems: maximize objective subject to scarcity constraints

- Examples
  - Consumer choice
  - Social planning problems
  - Principal-agent problems
  - Life-cycle problems
  - Profit maximization
  - Portfolio choice
Equations are Fundamental in Economics Modeling

• Equilibrium: a collection of choices by economic actors that are consistent with scarcity and individual rationality

• Demand equals supply
  – Competitive equilibrium
  – Asset market equilibrium
  – Dynamic market equilibrium

• Nash-Cournot
  – Oligopoly theory
  – Games of incomplete information
  – Games of asymmetric information
  – Political games
All Economic Analysis Uses Optimization and Equations

• Analysis of economic data is an optimization problem

• Unknown parameters are chosen so as to maximize the compatibility between statistical model and data
  – Least squares methods
  – Method of moments
  – Maximum likelihood

• Unknown parameters are chosen to fit data and satisfy equilibrium conditions
  – Structural estimation
  – A constrained optimization problem
Numerical Analysis is Applied Economics

- Numerical analysis is the development of computational tools that best use scarce computational resources to accomplish a computational task

- Scarce resources
  - Computer time
  - Programmer time
  - Programmer ability

- Objective
  - Accuracy
  - Speed

- Technologies
  - Memory
  - Processor
  - Communication links
Computation is About Using Computers

- You need to understand what computers do
  - Numbers - stored with infinite precision
  - Operations - executed with small errors
  - Storage methods and cache management
  - Interpreted versus compiled code

- As computer technologies change, the choice of algorithms changes
  - Single precision to double precision
  - Expensive memory to cheap memory
  - Serial to parallel processing (e.g., GPUs)
Progress in Hardware

- Moore’s law for semiconductors (Moore gives Moore’s law about another 10-15 years)
- Optical computing
- Quantum computing

![Graph showing trends in computation speed: flops vs. year]

Figure 1: Trends in computation speed: flops vs. year

- Economists usually constrain themselves to using personal computers.
Hardware and Algorithms: Substitutes or Complements?

- Typical economist view: speed allows you to avoid learning about computation and methods

- Computational mathematicians’ view
  - There are many possible methods, varying in fixed costs and marginal costs
  - Faster computers make it rational to invest in the fixed costs
    * Development costs
    * Use high fixed-cost methods that reduce marginal costs
  - Hardware speed and algorithm development are complements

- Historical pattern
  - Speed doubles every two years.
  - Algorithm efficiency grows at a similar, sometimes faster, rate
Objectives of ICE10

- Acquaint PhD students with the current state-of-the-art algorithms and software.
- Teach the basic concepts behind algorithm development, now and in the future.